Medici Family Influence

Various Measures of Centrality

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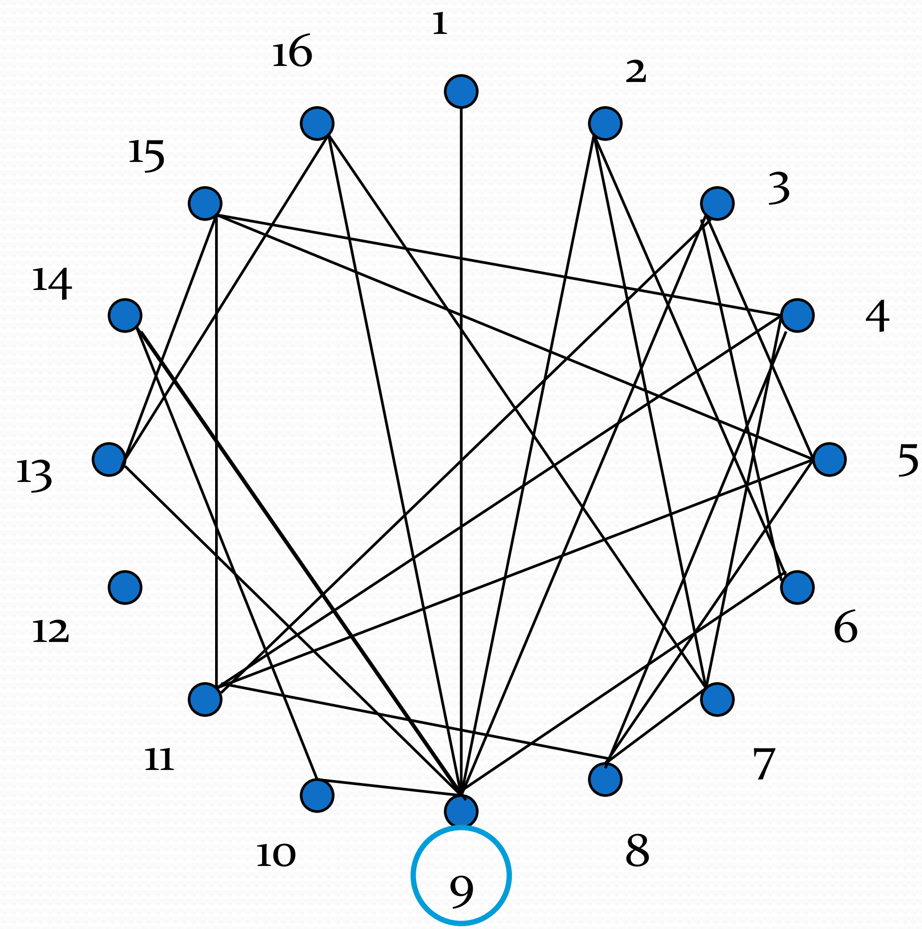
Matrix of Family Relationships

F 1	F 2	F 3	F4	F5	F6	F7	F8	F9*	F10	F11	F13	F14	F15	F16
0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
0	0	0	0	0	1	1	0	1	0	0	0	0	0	0
0	0	0	0	2	1	0	0	2	0	1	0	0	0	0
0	0	0	0	0	0	2	1	0	0	2	0	0	1	0
0	0	1	0	0	0	0	1	0	0	2	0	0	1	0
0	0	1	1	0	0	0	0	1	0	0	0	0	0	0
0	1	0	2	0	0	0	2	0	0	0	0	0	0	1
0	0	0	1	1	0	2	0	0	0	1	0	0	0	0
1	1	2	0	0	1	0	0	0	1	0	1	2	0	2
0	0	0	0	0	0	0	0	1	0	0	0	1	0	0
0	0	1	2	2	0	0	1	0	0	0	0	0	1	0
0	0	0	0	0	0	0	0	1	0	0	0	0	1	1
0	0	0	0	0	0	0	0	2	1	0	0	0	0	0
0	0	0	1	1	0	0	0	0	0	1	1	0	0	0
0	0	0	0	0	0	1	0	2	0	0	1	0	0	0

* Family number 9 is the Medici Family

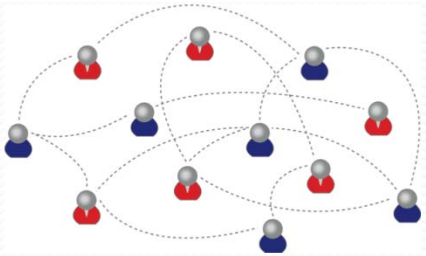
Map of Connections



Degree

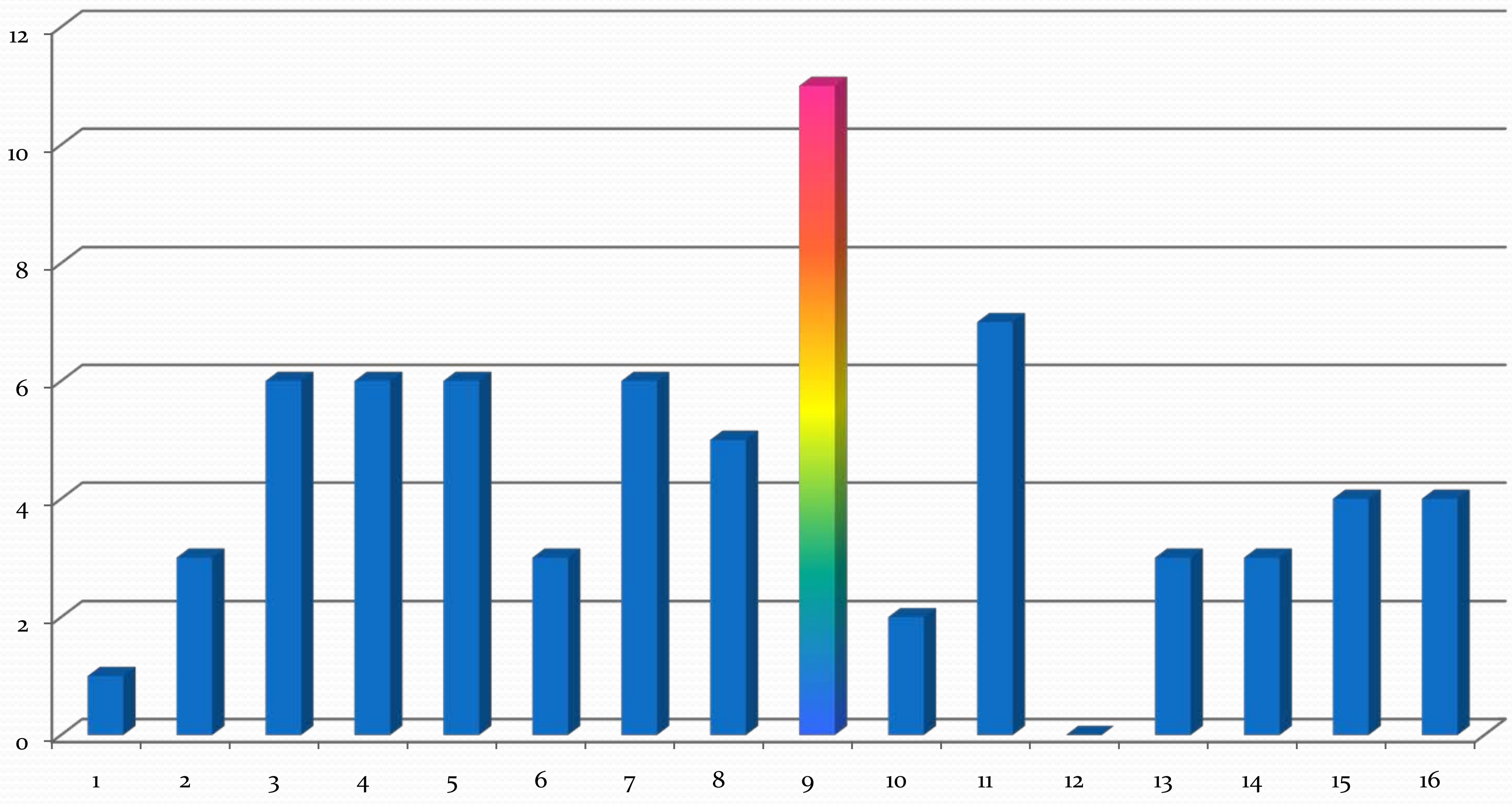
- Degree of each family is a simple way of determining centrality
- To find the number of degree per node (or family), count how many relationships there are between that node and the other nodes.
- For Example, the Medici Family has a degree of 11

(The Medici Family is node number 9)



Degree of Each Family

=number of links per node

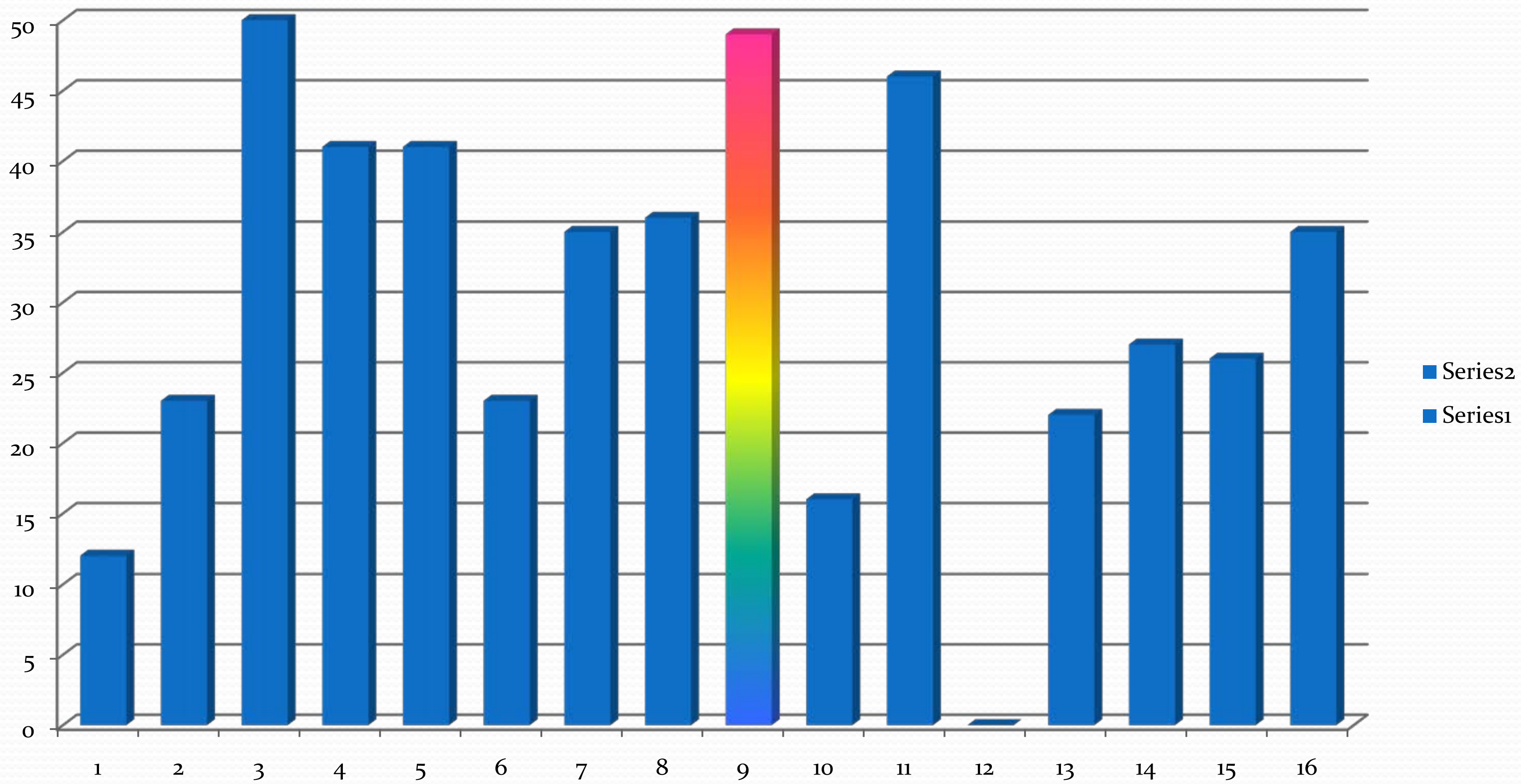


Power

- A slightly more complicated way of calculating centrality
- Takes into account people two relationships away
- (like a friend of a friend)
- Take the matrix M(people away a distance of 1) and add it to M*M(people away an exact distance of 2).
- Sum each row in the resulting matrix to obtain the power

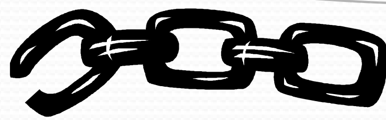
Power of Each Family

=M+M^2



In this case, Family number 3 has the most power

Markov Chains

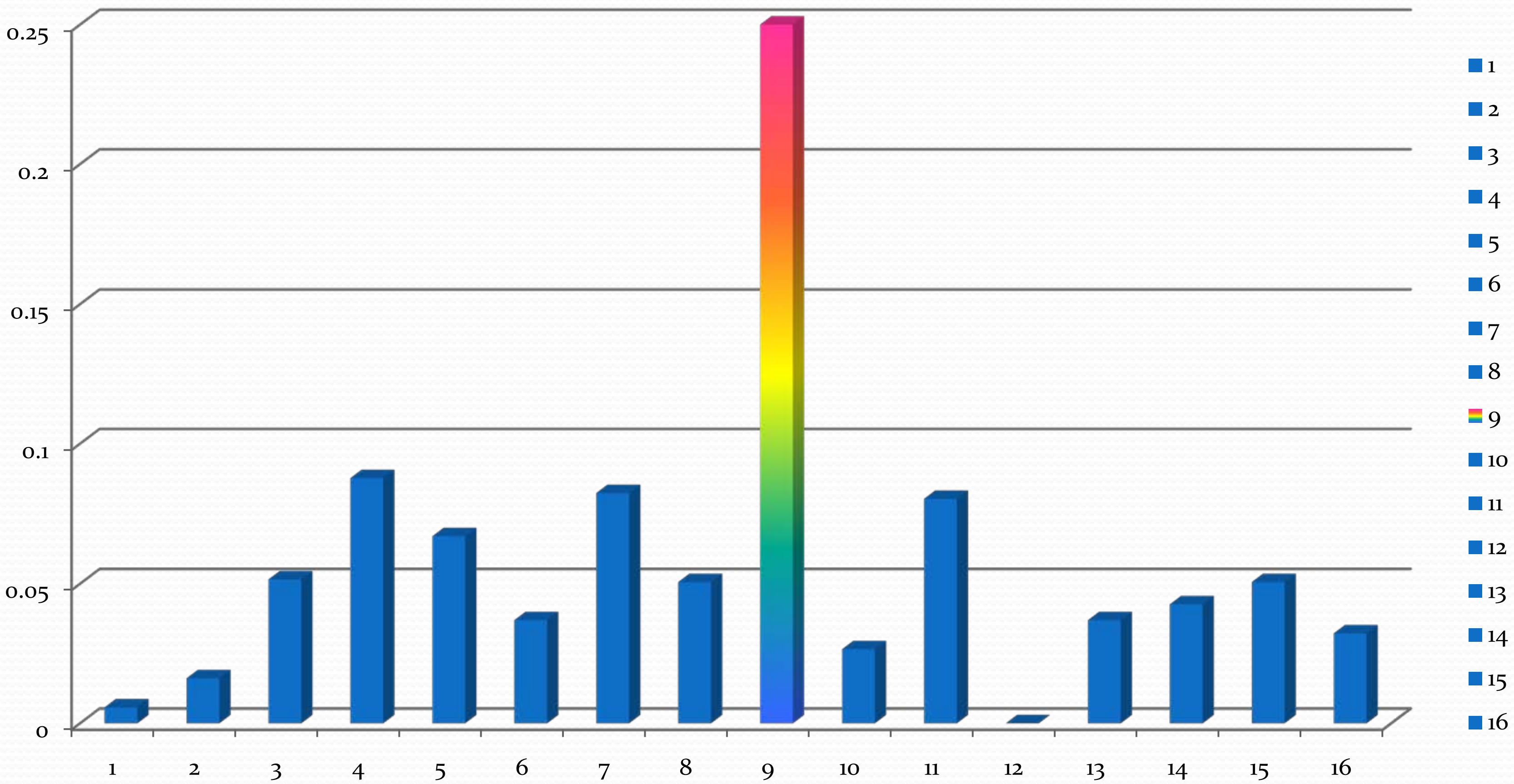


- Illustrates transition diagrams of probability
- Rows add up to 1
- To transform an adjacency matrix into a Markov chain, divide each element in a row by the row total
- This can be used to find the total fraction of influence of each family
- From now on, we will exclude family 12. It is isolated from all of the other families.

Matrix of Markov Chain of Family Relationships

F 1	F 2	F 3	F4	F5	F6	F7	F8	F9	F10	F11	F13	F14	F15	F16
0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
0	0	0	0	0	1/3	1/3	0	1/3	0	0	0	0	0	0
0	0	0	0	1/3	1/6	0	0	1/3	0	1/6	0	0	0	0
0	0	0	0	0	0	1/3	1/6	0	0	1/33	0	0	1/6	0
0	0	1/6	0	0	0	0	1/6	0	0	1/33	0	0	1/6	0
0	0	1/3	1/3	0	0	0	0	1/3	0	0	0	0	0	0
0	1/6	0	1/3	0	0	0	1/3	0	0	0	0	0	0	0
0	0	0	0.2	0.2	0	0.4	0	0	0	0.2	0	0	0	0
1/11	1/11	2/11	0	0	1/11	0	0	0	1/11	0	1/11	2/11	0	1/11
0	0	0	0	0	0	0	0	0.5	0	0	0	0.5	0	0
0	0	1/7	2/7	2/7	0	0	1/7	0	0	0	0	0	1/7	0
0	0	0	0	0	0	0	0	1/3	0	0	0	0	1/3	1/3
0	0	0	0	0	0	0	0	2/3	1/3	0	0	0	0	0
0	0	0	0.25	0.25	0	0	0	0	0	0.25	0.25	0	0	0
0	0	0	0	0	0	0.25	0	0.5	0	0	0.25	0	0	0

Fraction of Influence per Family



Package Problem

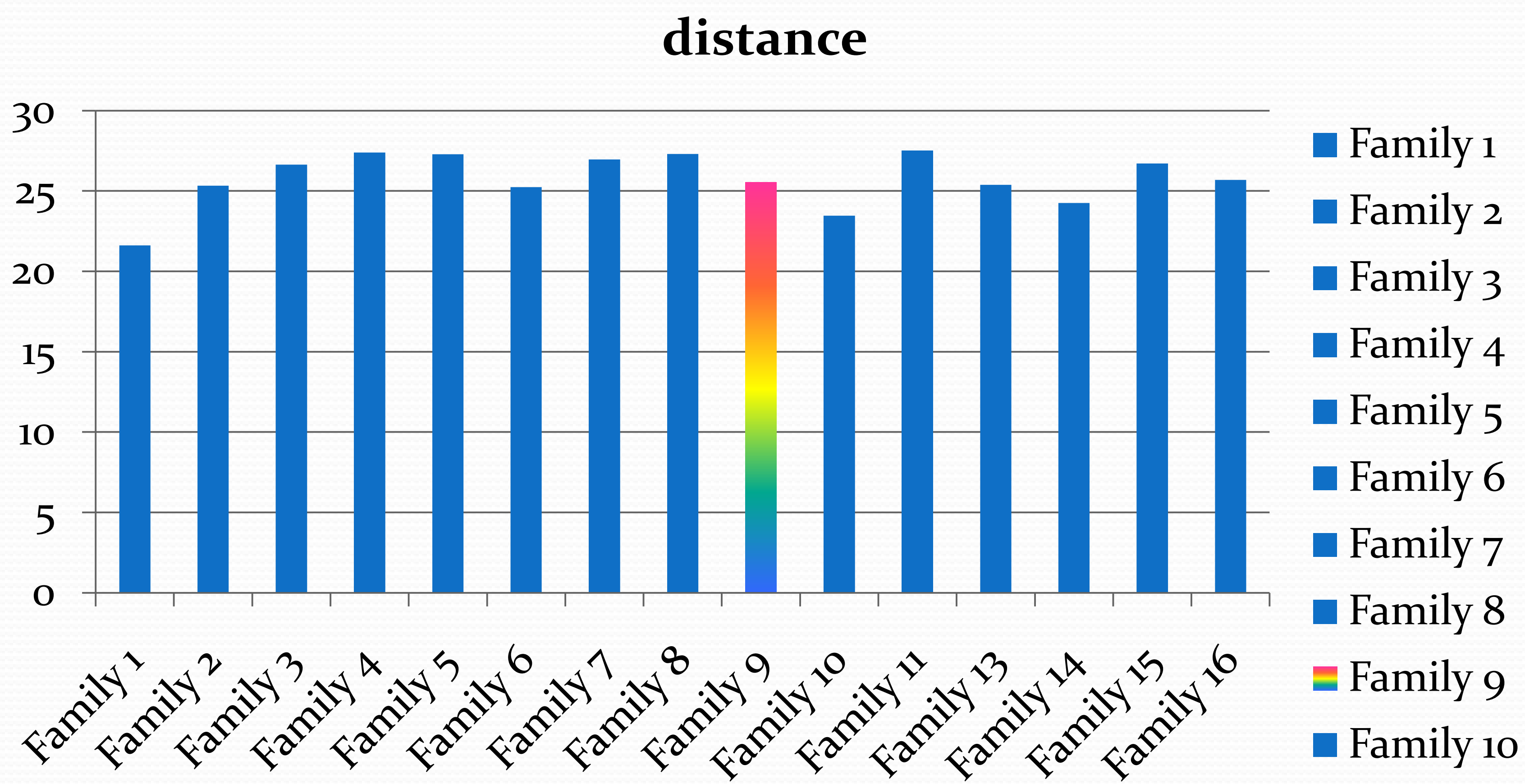
Say one family wants to send a secret package to another family, but the first family can only send the package through people that family knows. If the package is randomly passed family to family, how long on average will it take the package to get to the desired family?

Approach: Use a Markov chain matrix to determine the average number of transfers from one family.

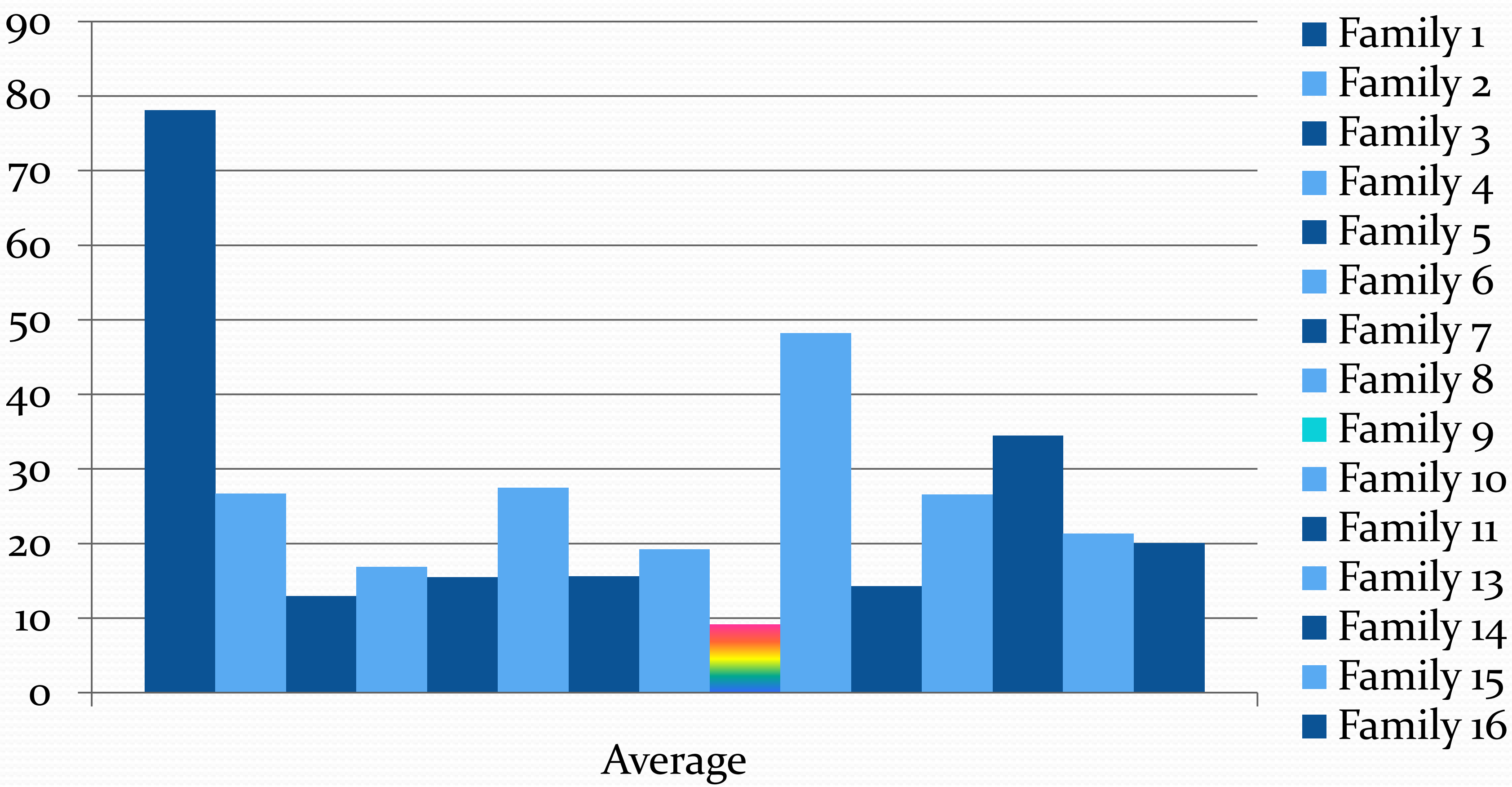
Average number of passes

F1	F2	F3	F4	F5	F6	F7	F8	F9	F10	F11	F13	F14	F15	F16
0	25	13	22	19	25	19	24	1	41	19	25	27	25	17
77	0	13	18	18	19	13	20	8	48	17	28	34	24	20
78	27	0	18	11	24	17	20	9	49	12	28	36	21	22
84	29	15	0	13	32	10	12	15	55	8	29	42	16	23
83	30	10	14	0	30	15	15	14	54	8	29	40	17	24
76	18	9	20	17	0	17	22	7	47	17	28	33	24	20
82	25	15	10	15	30	0	12	13	54	12	29	40	21	20
84	29	14	10	11	31	9	0	15	55	9	30	42	19	23
69	24	12	21	18	24	18	23	0	40	18	24	26	24	16
71	26	14	23	20	26	20	25	2	0	19	26	14	26	18
84	30	12	11	9	31	14	14	15	55	0	29	41	16	24
77	28	14	18	17	29	17	21	8	48	15	0	34	17	14
71	26	13	22	20	26	20	24	2	28	19	26	0	26	18
83	30	14	12	11	31	15	17	14	54	9	23	40	0	22
75	26	14	18	18	28	14	21	6	47	17	20	33	22	0

Average distance FROM each family



Average distance TO each family

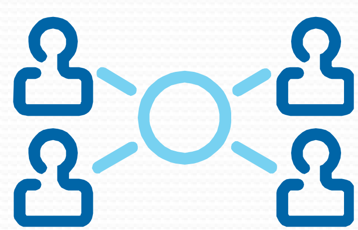


Minimum Distance to Each Family

F1	F2	F3	F4	F5	F6	F7	F8	F9	F10	F11	F13	F14	F15	F16
0	2	2	4	3	2	3	4	1	2	3	2	2	3	2
2	0	2	2	3	1	1	2	1	2	3	2	2	3	2
2	2	0	2	1	1	3	2	1	2	1	2	2	2	2
4	2	2	0	2	3	1	1	3	4	1	2	4	1	2
3	3	1	2	0	2	2	1	2	3	1	2	3	1	3
2	1	1	3	2	0	2	3	1	2	2	2	2	3	2
3	1	3	1	2	2	0	1	2	3	2	2	3	2	1
4	2	2	1	1	3	1	0	3	4	1	3	4	2	2
1	1	1	3	2	1	2	3	0	1	2	1	1	2	1
2	2	2	4	3	2	3	4	1	0	3	2	1	3	2
3	3	1	1	1	2	2	1	2	3	0	2	3	1	3
2	2	2	2	2	2	2	3	1	2	2	0	2	1	1
2	2	2	4	3	2	3	4	1	1	3	2	0	3	2
3	3	2	1	1	3	2	2	2	3	1	1	3	0	2
2	2	2	2	3	2	1	2	1	2	3	1	2	2	0

Family	Distance from F9
F1	1
F2	1
F3	1
F4	3
F5	2
F6	1
F7	2
F8	3
F9	0
F10	1
F11	2
F13	1
F14	1
F15	2
F16	1

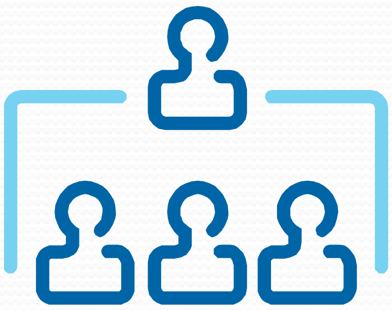
Family 9 would benefit most from forming connections with these families because they would minimize the distance between the two families. If a connection would be made with family 4 or 8, the shortest paths would be within a maximum of 2 links. The table on the right shows the minimum distance from family 9 after a connection to family 4 was added.



Family	Distance from F9
F1	1
F2	1
F3	1
F4	1
F5	2
F6	1
F7	2
F8	2
F9	0
F10	1
F11	2
F13	1
F14	1
F15	2
F16	1

Most Used Paths

- Big Question: What connections are used most frequently to obtain the shortest path?
- Approach: Using the shortest paths, count the number of times a connection is made.



Path Usage for Shortest Path (from row to column)

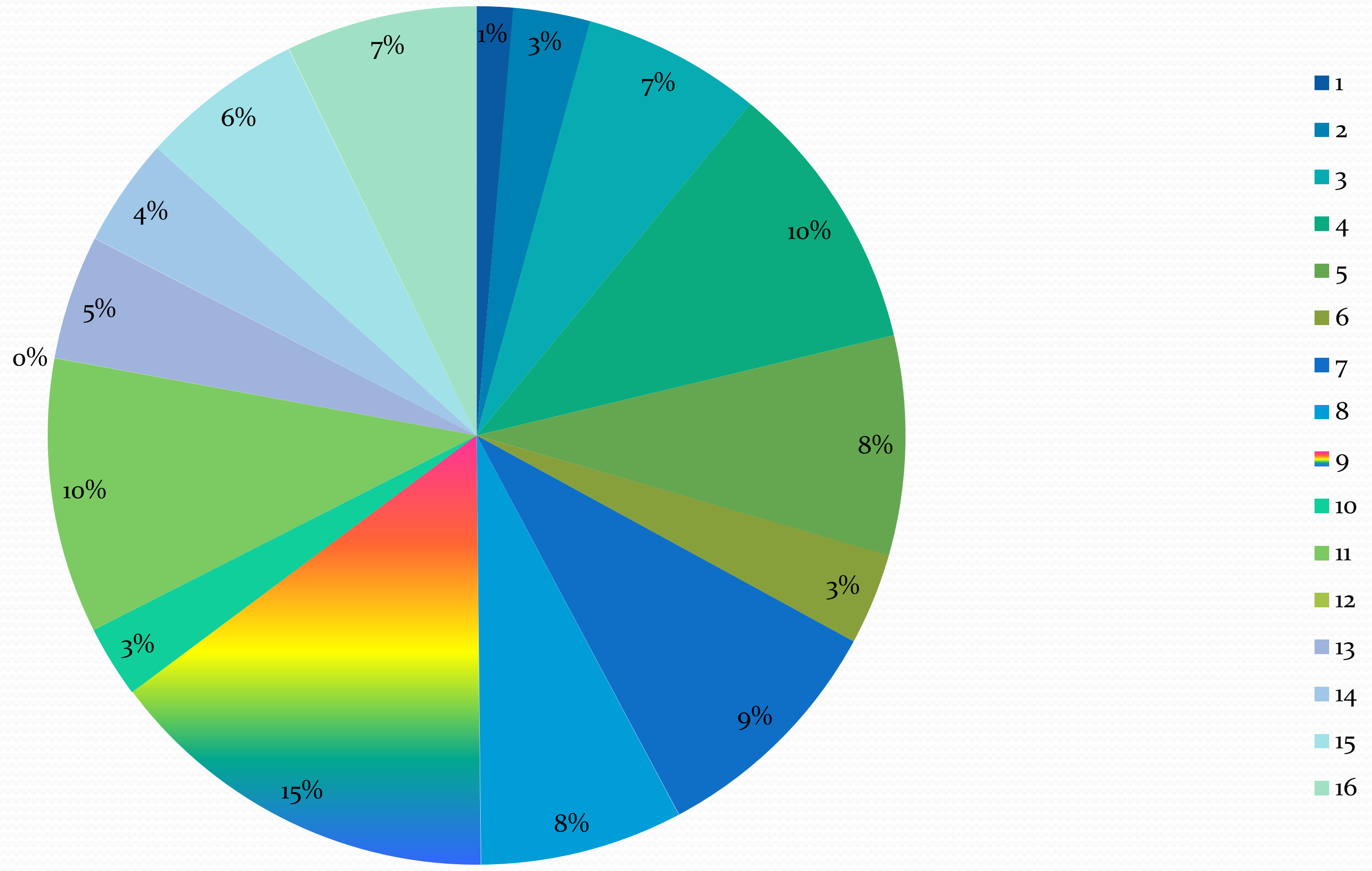
F1	F2	F3	F4	F5	F6	F7	F8	F9	F10	F11	F13	F14	F15	F16
0	14	14	13	12	13	13	12	12	11	11	15	11	13	14
13	0	15	12	13	17	15	11	13	12	12	16	12	14	15
14	16	0	13	15	18	15	12	14	13	14	17	13	13	16
12	12	12	0	8	13	12	11	9	11	10	13	11	12	12
12	14	15	9	0	11	9	11	7	11	10	13	11	12	14
16	18	21	18	14	0	15	14	16	15	13	19	15	15	18
13	16	15	13	9	12	0	12	8	12	8	14	12	10	16
11	11	11	11	10	12	11	0	8	10	9	14	10	8	11
13	15	15	9	8	14	9	10	0	12	7	16	12	9	15
11	13	13	12	11	12	12	11	11	0	10	14	13	12	13
11	13	14	11	10	10	8	10	6	10	0	12	10	11	13
15	17	17	14	13	16	14	15	15	14	12	0	14	17	20
11	13	13	12	11	12	12	11	11	13	10	14	0	12	13
12	14	12	12	11	13	9	8	7	11	10	16	11	0	12
14	16	16	13	14	15	16	12	14	13	13	20	13	13	0

The most used path is from Family 6 to Family 3

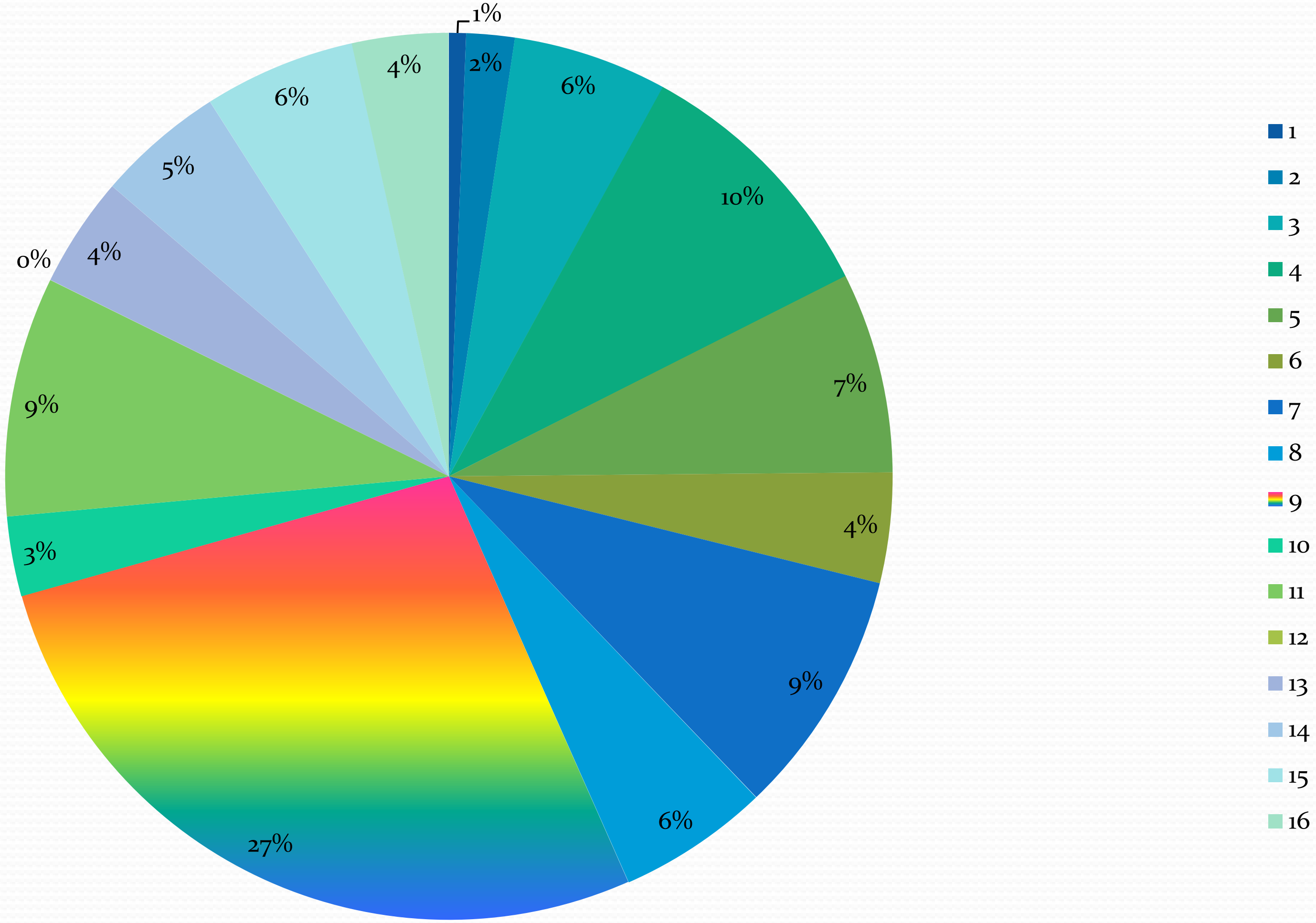
DeGroot Model

- This demonstrates what percent of a decision will belong to each family.
- Uses Markov chains to determine how a consensus will be reached.
- Solve $\prod * T = \prod$
- $\prod$= the static constant of the Markov chain

DeGroot percentage (own opinion excluded)



DeGroot Percentage
(considering own opinion as ½ of influence)

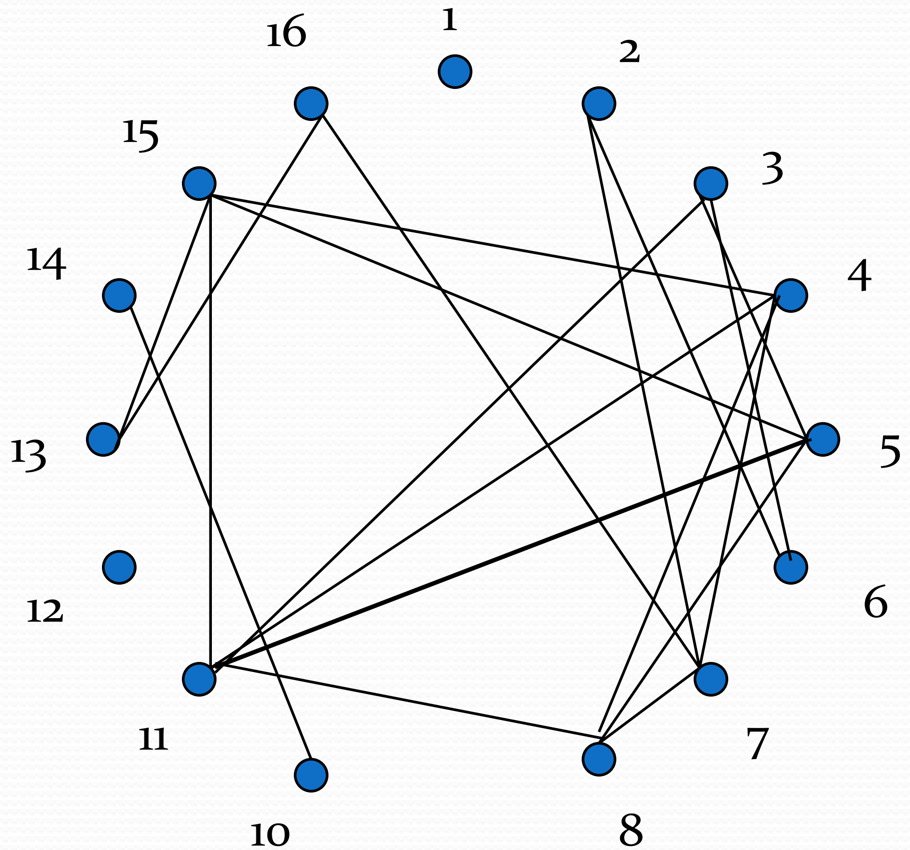


Life without the Medici

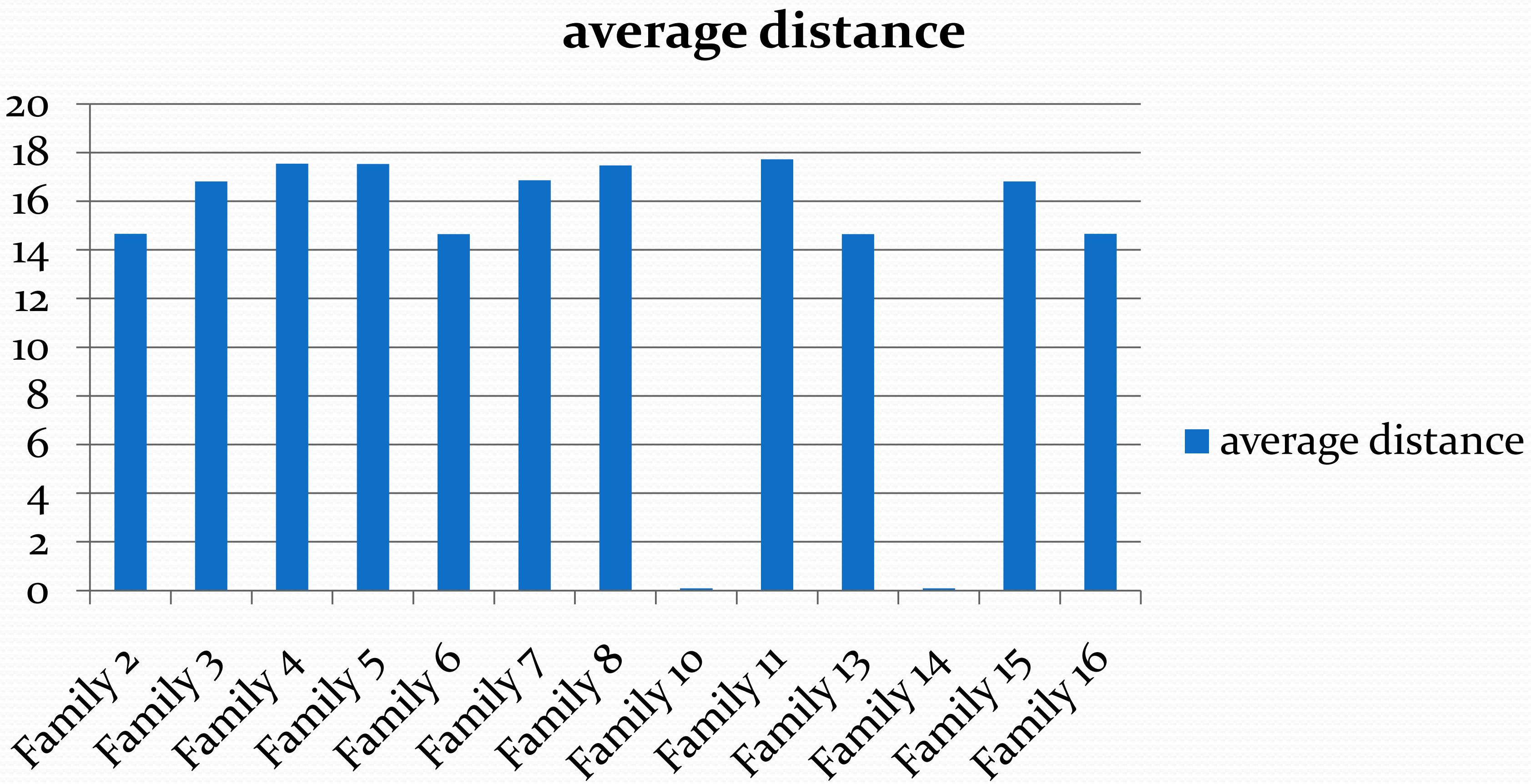
- Deleting the Medici Family form the Markov Chain yields:

F1	F2	F3	F4	F5	F6	F7	F8	F10	F11	F13	F14	F15	F16
0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0.5	0.5	0	0	0	0	0	0	0
0	0	0	0	0.5	0.25	0	0	0	0.25	0	0	0	0
0	0	0	0	0	0	0.333333	0.166667	0	0.333333	0	0	0.166667	0
0	0	0.333333	0	0	0	0	0.166667	0	0.333333	0	0	0.166667	0
0	0.5	0.5	0	0	0	0	0	0	0	0	0	0	0
0	0.166667	0	0.333333	0	0	0	0.333333	0	0	0	0	0	0.166667
0	0	0	0.2	0.2	0	0.4	0	0	0.2	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	1	0	0
0	0	0.142857	0.285714	0.285714	0	0	0.142857	0	0	0	0	0.142857	0
0	0	0	0	0	0	0	0	0	0	0	0	0.5	0.5
0	0	0	0	0	0	0	0	1	0	0	0	0	0
0	0	0	0.25	0.25	0	0	0	0	0.25	0.25	0	0	0
0	0	0	0	0	0	0.5	0	0	0	0.5	0	0	0

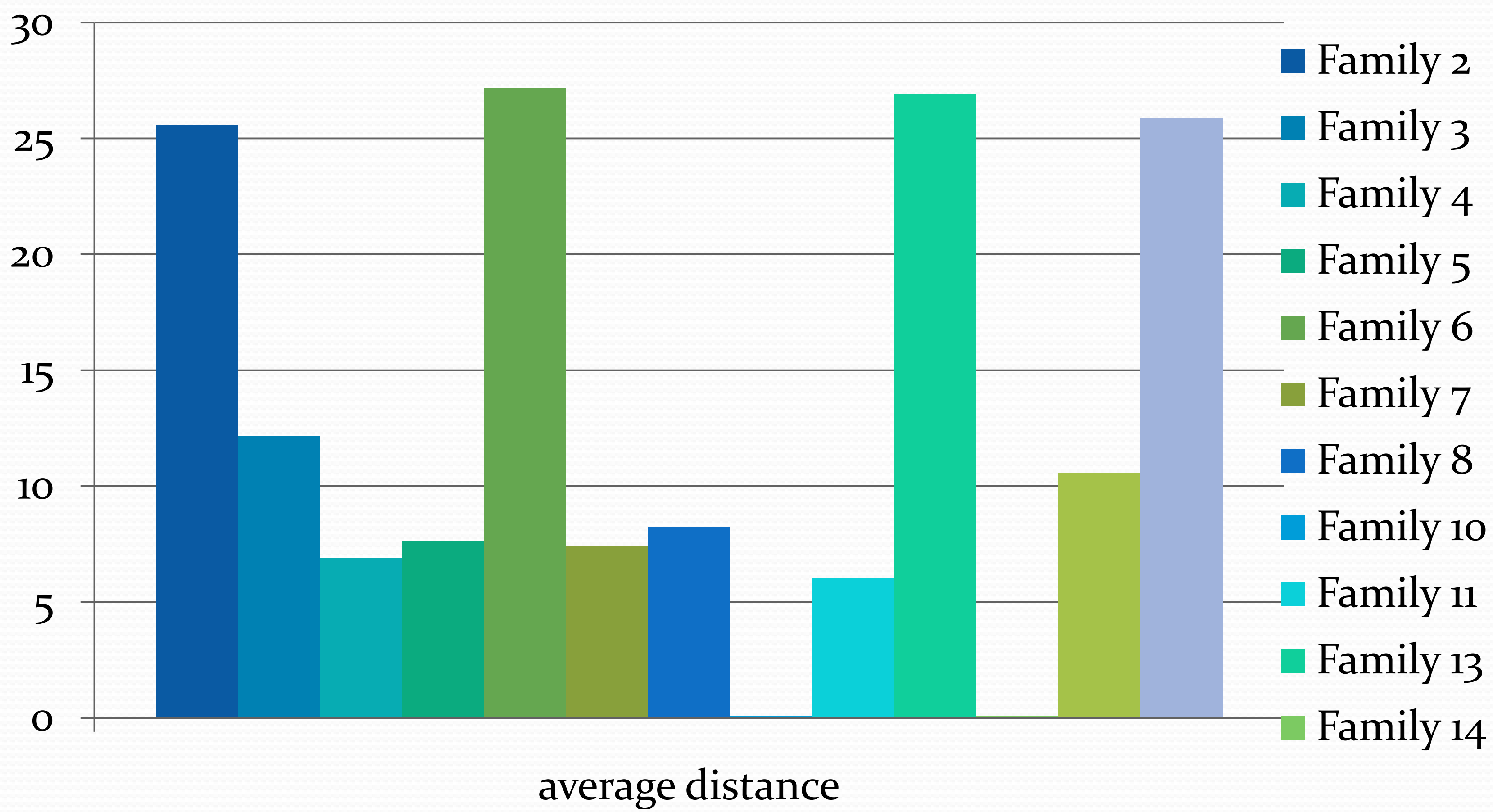
Graph of Relationships Without the Medici



Distance FROM each family (no Medici)



Distance TO each family (no Medici)



Without the Medici

- Family 1 becomes completely disconnected
- The group is no longer connected
- Families 10 and 14 are isolated from the rest of the families
- Packages cannot be sent between certain families
- Eleven links are destroyed
- Therefore, the Medici family is a critical point in the Renaissance family social group.

…And this all came from a single 16X16 Matrix

F 1	F 2	F 3	F4	F5	F6	F7	F8	F9*	F10	F11	F13	F14	F15	F16
0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
0	0	0	0	0	1	1	0	1	0	0	0	0	0	0
0	0	0	0	2	1	0	0	2	0	1	0	0	0	0
0	0	0	0	0	0	2	1	0	0	2	0	0	1	0
0	0	1	0	0	0	0	1	0	0	2	0	0	1	0
0	0	1	1	0	0	0	0	1	0	0	0	0	0	0
0	1	0	2	0	0	0	2	0	0	0	0	0	0	1
0	0	0	1	1	0	2	0	0	0	1	0	0	0	0
1	1	2	0	0	1	0	0	0	1	0	1	2	0	2
0	0	0	0	0	0	0	0	1	0	0	0	1	0	0
0	0	1	2	2	0	0	1	0	0	0	0	0	1	0
0	0	0	0	0	0	0	0	1	0	0	0	0	1	1
0	0	0	0	0	0	0	0	2	1	0	0	0	0	0
0	0	0	1	1	0	0	0	0	0	1	1	0	0	0
0	0	0	0	0	0	1	0	2	0	0	1	0	0	0